

Unit Topic:
 Logic, Proof and Triangles

Subject Area Essential Question (s):

- What is the purpose of having a defined mathematical language?
- What can we learn from studying the relationship between numbers, figures, and operations?
- How can we use data to construct reasonable expectations?
- What does it mean to think logically?
- What role has math played in the human experience?

Unit Level Essential Question(s):

- How can we prove theorems using deductive reasoning and the axioms of logic?
- What does it mean for two objects to be congruent?
- How can we prove that triangles are congruent?

Goals

A: Skills

Communicator: Receptive

- Identify the hypothesis and conclusion of a conditional statement.

Communicator: Expressive

- Write a proof using deduction or write an indirect proof.

Investigator: Inquiry

- Determine if a conditional is a biconditional.
- Determine the truth value of a logical statement.

Investigator: Innovation

- Use established mathematical methods and logical reasoning skills to prove the two triangles are congruent.
- Construct the converse, inverse, and contrapositive of a conditional statement.
- Solve problems using the Triangle Angle Sum theorem and the Exterior Angle theorem.

<p>B: Content Knowledge</p> <ul style="list-style-type: none"> • Triangle • Theorem • Hypothesis, Conclusion, Conditional, Biconditional, Contrapositive, Converse, Inverse • Exterior Angle • Deductive Reasoning • Inductive Reasoning • Parallel • Transversal • corresponding angles • alternate interior angles • consecutive interior angles • consecutive exterior angles 	<ul style="list-style-type: none"> • Congruent • Reflexive, Symmetric, Transitive P.o.C/P.o.E • Addition, Subtraction, Substitution P.o.E. • SAS, AAS, SSS, ASA Congruence Postulates • Triangle Angle Sum Theorem • Exterior Angle Theorem • Alternate exterior angles
---	--

MA State Frameworks

G-CO

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
3. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
4. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
5. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
6. Construct an equilateral triangle.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.

Write expressions in equivalent forms to solve problems.

2. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ➡
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities A-REI

1. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

1. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
2. Solve linear equations and inequalities in one variable involving absolute value.
3. Solve quadratic equations in one variable.
 - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

Solve systems of equations.

4. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

C: Understandings

Communicator: Receptive

- How to determine the underlying logical structure of spoken arguments.
- How to describe the relationships of a geometric figure (triangles) using algebraic expressions.

Communicator: Expressive

- How to write table proofs.
- How to draw Venn diagrams for true statements

Investigator: Inquiry

- How to determine whether two given triangles are congruent.
- How to determine the truth value of any statement.
- How to identify counterexamples for false statements.

Investigator: Innovation

- How to prove a logical statement using deduction.

Inquiry - and Skill-Based Teaching and Learning Ideas:

- Review necessary Algebra skills with a class discussion. Notice verbal response. Give independent worksheets, and vary according to a student's need for review.
- As a class, work through the proof scene in *To Kill a Mockingbird*, and translate the language into logical symbols and statements.
- Discuss as a whole class the logic and truth value of statements generated by the class.
- **Projects** where the students determine the criteria necessary to prove the congruence of triangles.
- **Test (or two quizzes)** - Assess students' retention of definitions and logical notations and rules, and their ability to apply these rules in various contexts.

Key Texts and Resources:

- CME Project *Geometry* (Chapter 2)
- Pearson's *Common Core Geometry*
- Sage
- GeoGebra
- Kuta Software